Link prediction

Supervised Random Walks: Predicting and Recommending Links in Social Networks

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Motivation

- Link prediction, link recommendation: predicting future links between existing nodes.

- Applications:
  - For social networks it has direct business consequences,
  - For networks in organizations, it suggests possible new collaborations.
Supervised framework

- For a given node $s$ positive examples are nodes to which $s$ is linked and negative examples are all the other nodes.

- This cannot be viewed as a classification task → imbalanced classes
  - For example in Facebook, nodes are in average linked to 100 other nodes while Facebook has more than 500 million existing nodes.

- Solution: Rank nodes instead of classifying them. Other popular methods for ranking nodes:
  - PageRank,
  - Random walks with restarts
  - The stationary distribution of such random walks assigns each node a score which gives a ranking of how close to the considered node are other nodes in the network.
Problem formulation

- Given a directed graph \( G(V, E) \), a node \( s \) and two sets: nodes to which \( s \) creates edges \( D = \{d_1, \ldots, d_k\} \) (destination nodes) and nodes to which \( s \) does not create edges \( L = \{l_1, \ldots, l_n\} \) (no-link nodes).

- Each edge \( (u, v) \in E \times E \) is characterized by a feature vector \( \Psi_{uv} \in \mathcal{X} \) that describes the nodes \( u \) and \( v \) (age, gender, hometown, etc.) and the interaction attributes (when the edge has been created, how many messages \( u \) and \( v \) exchanged, etc.).

- A function \( f_w : \mathcal{X} \rightarrow \mathbb{R}_+ \) estimating edge strengths is then learned upon \( D \cup L \). Considered functions:
  - Exponential edge strength: \( f_w(\Psi_{uv}) = e^{w \cdot \Psi_{uv}} \)
  - Logistic edge strength: \( f_w(\Psi_{uv}) = \frac{1}{1 + e^{-w \cdot \Psi_{uv}}} \)
Problem formulation

Find parameters $w$ so that the function $f_w$ assigns edge weights in a such way that the random walk will be more likely to visit nodes in $D$ than $L$

That is if $p$ is the vector scores, $\forall d \in D, \forall l \in L, p_l < p_d$

The proposed optimization problem

$$\min_w F(w) = \|w\|^2 + \lambda \sum_{s \in S} \sum_{d \in D_s, l \in L_s} h(p_l - p_d)$$

Where $h(p_l - p_d) = 0$ if $p_l < p_d$ and $h(p_l - p_d) > 0$ otherwise.

Remarks:

- The optimisation problem is not defined in a traditional ML way
- scores $p$ depend on edge strengths estimated by $f_w$. 
Dependency between $p$ and $w$

- Consider the random walk stochastic transition matrix $Q$,

$$\forall u, \forall v, Q_{uv} = (1 - \alpha) \frac{f_w(\Psi_{uv})}{\sum_z f_w(\Psi_{uz})} + \alpha 1_{v=s}$$

The *proposed* interpretation: $Q_{uv}$ is the conditional probability that a walk will traverse edge $(u, v)$ given that it is currently at node $u$. $\alpha \in [0, 1]$ is the restart probability: with probability $\alpha$ the random walk jumps back to seed node $s$ and restarts.

- The vector scores $p$ is the stationary distribution of the Random walk with restarts, and it is the solution of:

$$p^t = p^t Q$$
Optimization problem is solved using a simple gradient descent method

\[ \forall k, \frac{\partial F(w)}{\partial w_k} = 2w_k + \lambda \sum_{s \in S} \sum_{d \in D_s, l \in L_s} \frac{\partial h(x_{ld})}{\partial x_{ld}} \left( \frac{\partial p_l}{\partial w_k} - \frac{\partial p_d}{\partial w_k} \right) \]

Where \( x_{ld} = p_l - p_d \).

As \( \mathbf{p} \) is the principal eigenvector of matrix \( Q \), \( p_u = \sum_j p_j Q_{ju} \) so

\[ \frac{\partial p_u}{\partial w_k} = \sum_j Q_{ju} \frac{\partial p_j}{\partial w_k} + p_j \frac{\partial Q_{ju}}{\partial w_k} \]

Remark: \( p_u \) and \( \frac{\partial p_u}{\partial w_k} \) are recursively entangled \( \rightarrow \) the derivatives \( \frac{\partial p_u}{\partial w_k} \) are recursively estimated applying the chain rule.
Features $\Psi_{uv}$ for the co-authorship network

- Number of papers written by $u$ before $t$,
- Number of papers written by $v$ before $t$
- Number of papers $u$ and $v$ co-authored
- Cosine similarity between the titles of papers written by $u$ and titles of $v$’s papers
- Time since $u$ and $v$ last co-authored a paper
- Number of common friends between $u$ and $v$. 